



Laser Resonator and Beam Quality

1. Laser resonator and modes

A simple optical resonator consists of two mirrors. Its most significant character is the open sides, which always produce diffraction losses. The functions of a laser resonator are (a) to achieve optical feedback and to generate laser beam; and (b) to restrict laser beam and to form a coherent beam. The total propagation through one round trip in an optical resonator can be described mathematically by a propagation integral which will have the general form

$$E^{(1)}(x, y) = e^{-jkp} \iint_{\text{input plane}} K(x, y, x_0, y_0) E^{(0)}(x_0, y_0) dx_0 dy_0 \quad (1)$$

where k is the propagation constant at the carrier frequency of the optical signal; p is the length of one period or round trip; and the integral is over the transverse co-ordinates at the reference or input plane. The function \tilde{K} appearing in this integral is commonly called the propagation kernel, since the field $\tilde{E}^{(1)}(x, y)$ after one propagation step can be obtained from the initial field $\tilde{E}^{(0)}(x_0, y_0)$ through the operation of the linear kernel or "propagator" $\tilde{K}(x, y, x_0, y_0)$.

Fox and Li simulated the iterative round trips of a wavefront $\tilde{E}(x, y)$ in a resonator by using numerical computation. It was certified that the eigenmodes do exist for a given resonator. Each one of these eigenmodes after one round trip satisfies the round-trip propagation expression

$$\tilde{E}_{nm}^{(1)} = \iint \tilde{K}(x, y, x_0, y_0) \tilde{E}_{nm}^{(0)}(x_0, y_0) dx_0 dy_0 = \gamma_{nm} \tilde{E}_{nm}^{(0)}(x, y) \quad (2)$$

where $\tilde{E}_{nm}(x, y)$ is a set of eigenmodes and γ_{nm} is a corresponding set of eigenvalues.

For a lowest-order gaussian beam, the radial intensity variation with spot size ω_0 in cylindrical co-ordinates is given by

$$I(r) = \frac{2P}{\pi\omega_0^2} e^{-2r^2/\omega_0^2} \quad (3)$$

where P is the total power in an laser beam. Fig. 1 shows the gaussian beam distributions, where $I(0) = \frac{2P}{\pi\omega_0^2}$. Generally, the beam waist ω_0 is defined as the spot diameter at which the intensity is 13.6% ($=1/e^2$) of the central intensity.

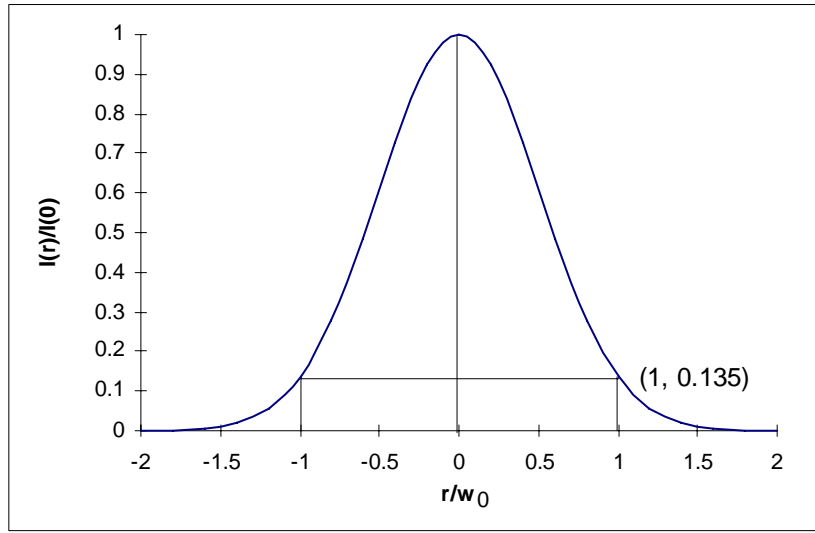


Fig. 1 Gaussian distribution and beam waist (*intensi.xls*)

For higher-order gaussian beam, the eignmodes can take the forms either of Hermite-gaussian functions in rectangular co-ordinates, or of Laguerre-gaussian functions in cylindrical co-ordinates. Any arbitrary paraxial optical beam $\tilde{E}_{nm}(x, y, z)$ in rectangular co-ordinates can be in the form

$$\tilde{E}_{nm}(x, y, z) = \sum \sum c_{nm} \tilde{u}_n(x, z) \tilde{u}_m^*(y, z) e^{-jkz} \quad (4)$$

$$c_{nm} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z) u_n^*(x, z) u_m^*(y, z) dx dy \quad (5)$$

$$\int_{-\infty}^{\infty} u_n^*(x, z) u_m(x, z) dx = \delta_{nm} \quad (6)$$

$$\tilde{u}_n(x, z) = \left(\frac{2}{\pi}\right)^{1/4} \left(\frac{1}{2^n n! \omega_0}\right)^{1/2} \left[\frac{\tilde{q}_0}{\tilde{q}_0^*} \frac{\tilde{q}^*(z)}{\tilde{q}(z)} \right]^{n/2} H_n\left(\frac{\sqrt{2}x}{\omega(z)}\right) \exp\left[-j \frac{kx^2}{2\tilde{q}(z)}\right] \quad (7)$$

$$\frac{1}{\tilde{q}(z)} = \frac{1}{R(z)} - j \frac{\lambda}{\pi \omega^2(z)} \quad (8)$$

where the H_n 's are the Hermite polynomials of order n , e^{-jkz} is phase shift term, $k = w/c$, w is resonance frequency, c is light speed, $R(z)$ is spherical curvature, and $\omega(z)$ is spot size.

Any arbitrary paraxial optical beam $\tilde{E}(r, \theta, z)$ in cylindrical co-ordinates can also be given in a similar form. If the output beam from a laser oscillating in one of these higher-order modes is projected onto a screen, the intensity patterns of various higher-order mode appear in a certain shapes. Figure 6 shows transverse mode patterns for Hermite-gaussian modes and Laguerre-gaussian modes of various orders.

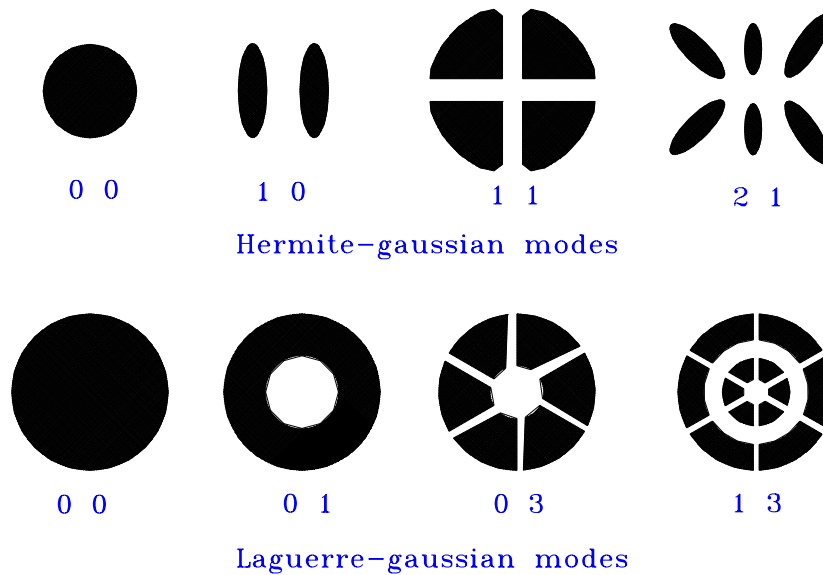


Fig. 2 Transverse mode patterns

If the spot size of TEM₀₀ mode, ω_0 , is known, the spot sizes of higher-order modes in rectangular co-ordinates can be given as

$$\omega_m = \sqrt{2m+1}\omega_0 \quad (9)$$

and

$$\omega_n = \sqrt{2n+1}\omega_0 \quad (10)$$

where m and n are the null numbers of x and y axes of modes.

The spot sizes of higher-order modes in cylindrical co-ordinates are given in Table 1.

Table 1 Spot sizes for high-order modes in cylindrical co-ordinates

Mode	TEM ₀₀	TEM ₁₀	TEM ₂₀	TEM ₀₁	TEM ₁₁	TEM ₂₁
Spot size	ω_0	$1.50\omega_0$	$1.77\omega_0$	$1.92\omega_0$	$2.21\omega_0$	$2.38\omega_0$

It is noted that the practical laser beams often behave in a combination of various modes, except special resonators or other ways used to obtain TEM₀₀ mode.

2. Laser Beam Quality

One of the most important properties of a laser resonator is the highly collimated or spatially coherent nature of the laser's output beam. The spatial beam quality of the output beam, namely beam diameter and propagation factor, is a critical parameter in a wide range of practical laser applications. This is because the spatial beam quality determines how tightly the beam can be focused or how well the beam propagates over long distances without significant spreading. For example, the output power from an ordinary propane torch can be many kilowatts, which are far higher than the power available from most lasers. Despite the high power, such a torch would be completely incapable of either cutting or welding steel plates. In contrast, a 500 W CO₂ laser can easily cut or weld steels. The propagation behaviours of the laser beam are responsible for this difference. Therefore, it was investigated by many authors in previous years on how to define and on how to measure the laser beam quality. An ISO Working Draft Committee has also been organised to set-up a standard for definitions and test methods of the laser beam quality.

The quality of a laser beam can have different meanings for different applications. In laser drilling, the most important quality characteristic could be the peak power. Therefore, the higher the peak power, the better the beam quality from the user's point of view. The gaussian beam distribution is also preferred for this type of applications. However, in heat treatment, the more uniform the

intensity distribution, the better the beam quality. Thus, a multi-mode laser beam is preferred. For most users, their concern is the performance of the beam on the workpiece. The key parameters of a laser beam are the effective minimum beam diameter (or beam waist), its location relative to the surface of the workpiece, and the beam divergence angle or the related propagation factor.

(1) Beam Divergence

As it is widely known, a laser beam expands due to diffraction spreading as it propagates away from the waist region. For an ideal TEM₀₀ gaussian beam, the divergence angle is easily calculated if the beam waist is given. For non-fundamental gaussian beam, the beam divergence can be calculated once the beam spot size at the focal plane has been measured. Dividing the spot size by the focal length, the approximate beam divergence θ (full angle) is given by

$$\theta = \frac{D_F}{F} \quad (11)$$

where F is the focal length of the focusing lens, and D_F is the beam diameter at the focal plane.

Generally speaking, the smaller the divergence, the better the beam quality. A small divergence means that the beam has a small diameter at a distance far away from the beam waist. The larger distance allowed between the focusing lens and the workpiece eliminates possible contamination to the focusing lens during laser processing.

(2) Beam Propagation Factor M^2

There are two major definitions to describe the beam quality: beam parameter product and beam propagation factor M^2 . Although the two parameters are different in defining the beam quality, the core ideas that the beam quality should indicate beam parameters and propagation characteristics are similar, and the defined beam qualities are correlated to each other.

The beam parameter product is defined to be the product of divergence and beam spot size. The argument is that reducing the divergence by using a beam expander would increase the beam spot size. Thus,

$$BPP = \frac{D\theta}{4} \quad (12)$$

where BPP is the beam parameter product, θ is the far-field full divergence angle, and D the near-field diameter (plane wave front).

The lower limit of this product is obtained for the fundamental mode with $BPP = \lambda/\pi$. For Nd:YAG lasers, the limit is 0.32 mrad.mm. For commercial systems, typical values are in the range of 20 - 40 mrad.mm.

The beam parameter product is important because it provides an indication of the focused beam size and the focal depth. For example, to efficiently couple the laser beam into a fibre, the following condition must be fulfilled

$$\frac{D\theta}{4} < \frac{d'}{2} NA \quad (13)$$

where d' is the fibre core diameter and NA is the numerical aperture of the fibre. Therefore the beam parameter product expresses exactly the requirement on NA .

In recent years, the laser community is more and more inclined to use the beam propagation factor, M^2 , to define laser beam quality. M^2 is the factor by which the divergence is greater than the perfect gaussian mode of the same minimum beam width. The focused spot size of the multimode beam with M^2 will be M times what would be the theoretical spot size of a perfect gaussian beam.

For the gaussian beam, the gaussian spot size $\omega(z)$ at any plane z perpendicular to the beam propagation direction expands with distance in free space in the form

$$\omega^2(z) = \omega_0^2 + \left(\frac{\lambda}{\pi\omega_0} \right)^2 (z - z_0)^2 \quad (14)$$

where z_0 is the beam waist location and $\omega_0 (=0.5D_0)$ is the beam radius as shown in Figure 3.

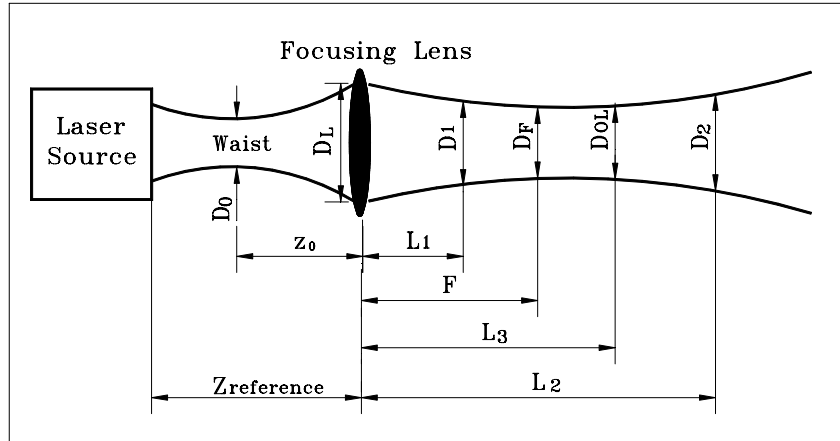


Figure 3 Some parameters associated with beam propagation

Similarly, for the arbitrary and possibly highly non-gaussian real laser beams, the spot size measured along the x and y directions, $\omega_x(z)$ and $\omega_y(z)$, at any plane z in free space will be given by

$$\omega_x^2(z) = \omega_{0x}^2 + \left(\frac{\lambda M_x^2}{\pi\omega_{0x}} \right)^2 (z - z_{0x})^2 \quad (15)$$

and

$$\omega_y^2(z) = \omega_{0y}^2 + \left(\frac{\lambda M_y^2}{\pi\omega_{0y}} \right)^2 (z - z_{0y})^2 \quad (16)$$

where ω_{0x} and ω_{0y} are the waist spot sizes, z_{0x} and z_{0y} are beam waist locations, and M_x^2 and M_y^2 are the beam propagation factors. Here x and y are co-ordinates of the horizontal and vertical axes.

Figure 3 illustrates the parameters involved in laser beam propagation. The beam propagation factor M^2 is calculated by

$$M^2 = \frac{\pi D_{OL}^2}{2\lambda(L_2 - L_1)} \left(\frac{D_1^2}{D_{OL}^2} - 1 \right)^{\frac{1}{2}} \quad (17)$$

For the common case where the lens is within one-third of a Rayleigh range of the beam waist, the calculation of M^2 within 5% can be simplified as

$$M^2 = \frac{\pi D_L D_F}{4\lambda F} \quad (18)$$

where F is the focal length of the known lens, D_F is the beam diameter at the focal plane, D_L is the beam diameter at the lens, and the other symbols are shown in Figure 3. The Rayleigh range, Z_R , is given by

$$Z_R = \frac{\pi\omega_0^2}{\lambda} \quad (19)$$

Substituting Equation (11) into Equation (18), we obtain

$$M^2 = \frac{\pi D_L \theta}{4\lambda} \quad (20)$$

Clearly, M^2 represents the influence of wavelength, beam diameter, and divergence angle on the beam quality. Table 2 gives some M^2 values for various laser beam modes. It is obvious that the smaller the M^2 value is, the better the beam quality will be.

Table 2 M^2 of some laser modes

Mode	TEM ₀₀	TEM ₀₁	TEM ₁₀	TEM ₁₁	TEM ₂₀	TEM ₂₁
M^2	1.0	2.0	3.0	4.0	5.0	6.0

From Equations (14) - (16), it can be concluded that the beam diameter at any plane z can be expressed in a binomial as

$$d^2 = A + Bz + Cz^2 \quad (21)$$

where d is the beam diameter, z is the measured position, and A , B , and C are the constants to be determined. Then the beam waist position z_0 , waist size ω_0 , and M^2 can be calculated by the following equations

$$z_0 = -\frac{B}{2C} \quad (22)$$

$$\omega_0 = \sqrt{A - \frac{B^2}{4C}} \quad (23)$$

and

$$M^2 = \frac{\pi}{4\lambda} \sqrt{AC - \frac{B^2}{4}} \quad (24)$$

Based on Equations (21) - (24), one needs only to measure the beam diameters at three selected z values along the beam and then fit the resulting spot sizes to the quadratic expressions in Equation (21). From the coefficients of this fit, the beam propagation factor M^2 can be calculated. However, in order to get good accuracy, the measurements were usually made at a large number of z values and the resulting data are best-fitted to the quadratic equation shown in Equation (21).

It should be pointed out that, in an alternative definition of the propagation factor, the reciprocal of M^2 is used such that an imperfect beam cannot have a number greater than unity.

3 Beam Diameter Measurement Methods

As already shown in the above sections, the beam diameter is a key factor in determining the beam quality. Therefore the measurement of the beam diameter is very important. There are a number of methods or algorithms for measuring the beam diameter, e.g. slit scan method, variable-aperture method, knife-edge scan method, and second-moment method. All the methods are reliable for perfect gaussian beams but suffer from various errors when applied to non-gaussian beam shapes. At present, research is being conducted into the use of the second moment method as a standard method of measuring beam width when using 2D beam profile data (e.g. CCD cameras). The results so far are promising. But it has difficulties in implementation since noise in the wings of the beam contributes excessive errors, resulting in a wide range of deviations in the beam width calculation. However, this problem can be solved mathematically.

(1) Slit Scan Method

The method uses a slit, which is about 1/20 the width of the beam to be measured so as to avoid slit convolution corrections. The method is to find the width between the points where the irradiance has reached 13.5% ($=1/e^2$) of the maximum irradiance. This is the beam diameter at $1/e^2$ intensity. One major advantage of this method is that mechanical slits attenuate the beam power very nicely. A disadvantage of the technique is that if the beam to be measured is extremely small, it becomes difficult to find slits sufficiently narrow to neglect the convolution error.

(2) Variable Aperture Method

The method requires that the beam be circular to within 1.15:1. The variable aperture must be aligned to the beam centre to within 10% of the beam diameter. The beam diameter is then the diameter which contains 86.5% of the total energy.

(3) Knife Edge Scan Method

There are two methods of measurement. The first finds the distance between the 10% and 90% energy levels; the distance determines a width which is equal to the $1/e^2$ width divided by 1.56. The second finds the distance between the 2.3% and 97.7% energy levels; no multiplication is required. The slit scan method produces larger corrections when compared to the variable-aperture method. An advantage of this technique is that one needs only a razor blade, a translation stage, and a uniform photocell/power meter to take measurements.

(4) Second Moment Method

The method requires the acquisition of a complete array of beam intensities using a raster-scanned pinhole, a vidicon, or a semiconductor charge-coupled device. For simplicity, it is assumed that the beam profile for the measurement of the beam diameter is acquired as a 256×256 square pixel array. Then, the first order linear moments x_c and y_c (beam centroid) are given by

$$x_c = \frac{1}{P_H} \sum_0^{255} \sum_0^{255} xP(x, y) \quad (25)$$

and

$$y_c = \frac{1}{P_H} \sum_0^{255} \sum_0^{255} yP(x, y) \quad (26)$$

where $P(x, y)$ is the local intensity at each pixel site and P_H is given by

$$P_H = \sum_0^{255} \sum_0^{255} P(x, y) \quad (27)$$

By calculating the second moments about the centroid, beam widths can be determined as

$$\langle x^2 \rangle_c = \frac{1}{P_H} \sum_0^{255} \sum_0^{255} (x - x_c)^2 P(x, y) \quad (28)$$

$$\langle y^2 \rangle_c = \frac{1}{P_H} \sum_0^{255} \sum_0^{255} (y - y_c)^2 P(x, y) \quad (29)$$

The beam diameters are given by

$$d_{ox} = 4\sqrt{\langle x^2 \rangle_c} \quad (30)$$

and

$$d_{oy} = 4\sqrt{\langle y^2 \rangle_c} \quad (31)$$

All of the above integrals should be truncated by integrating above the background level, otherwise erroneously large values for beam diameters can result. At present, there are many types of commercial laser beam diagnostics systems that can be used to measure the M^2 parameter and beam diameters.