



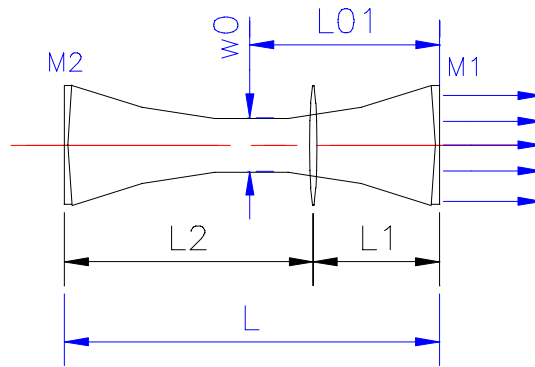
## Optical Calculation in Laser Marking System

In a typical laser marking system, optics consists of laser resonator (laser head), a beam expander, and a focal lens (f-theta lens).

### 1. Laser Beam of a Laser Resonator

#### 1.1 Theoretical Consideration

The resonator consists of two mirrors M1 and M2 with radii of curvature of  $R_1$  and  $R_2$  respectively. The thermal effects of the Nd:YAG rod are represented by a thermal lens with a focal length  $f_0$ , as shown in Fig. 1 .



*Fig. 1 Optical resonator*

The pertinent parameters of a resonator equivalent to one with an internal thin lens are

$$g_1 = 1 - \frac{L_2}{f_0} - \frac{L_0}{R_1} \quad (1)$$

$$g_2 = 1 - \frac{L_1}{f_0} - \frac{L_0}{R_2} \quad (2)$$

$$g = g_1 g_2 \quad (3)$$

where  $L_0 = L_1 + L_2 - (L_1 L_2 / f_0)$ ,  $L_1$  and  $L_2$  are the spacings between the thermal lens and mirrors M<sub>1</sub> and M<sub>2</sub> respectively.

In the resonator, the radius  $\omega_0$  of the beam waist and the distance  $L_{01}$  between the beam waist and mirror M<sub>1</sub> can be expressed as a function of the equivalent resonator parameters, i.e.

$$\omega_0^2 = \frac{\lambda L_0 [g_1 g_2 (1 - g_1 g_2)]^{\frac{1}{2}}}{\pi [g_2 (L_0 / R_1)^2 + g_1 (1 - g_1 g_2)]} \quad (4)$$

$$\text{and } L_{01} = \frac{L_0^2 g_2 / R_1}{g_2 (L_0 / R_1)^2 + g_1 (1 - g_1 g_2)} \quad (5)$$

The radii of the beam spot size at M1 and M2 are

$$\omega_1^2 = \frac{\lambda L}{\pi} \left[ \frac{g_2}{g_1 (1 - g_1 g_2)} \right]^2 \quad (6)$$

$$\text{and } \omega_2^2 = \frac{\lambda L}{\pi} \left[ \frac{g_1}{g_2 (1 - g_1 g_2)} \right]^2 \quad (7)$$

and the full-apex divergence angle in the far field is

$$\theta = \frac{2\lambda}{\pi \omega_0} \quad (8)$$

## 1.2 Calculations on Laser Resonator

As an example, following is a typical laser resonator with

L	:	700 mm
L1	:	300 mm or 350 mm
L2	:	L - L1
f <sub>0</sub>	:	300, 350, 400, 450, or 500 mm (For the $\phi 4 \times 100$ mm YAG rod, the thermal focal lengths range from 378 mm to 461 mm corresponding to input electrical power from 4 kW to 3.4 kW.)

We have the following results:

- (1)  $R_1 = \infty, R_2 = -1.5 \text{ m}, f_0 = 455 \text{ mm}, L1 = 300 \text{ mm}$   
 $\theta = 1.12 \text{ mrad}, \omega_0 = 0.6 \text{ mm}, g = 0.07$
- (2)  $R_1 = \infty, R_2 = -2.5 \text{ m}, f_0 = 455 \text{ mm}, L1 = 300 \text{ mm}$   
 $\theta = 1.19 \text{ mrad}, \omega_0 = 0.57 \text{ mm}, g = 0.06$

## 2. Beam Expander

The laser beam enter a beam expander after the laser beam is generated in the laser resonator. The most common type of beam expander is derived from the Galilean telescope which usually has one negative input lens and one positive output lens. The input lens presents a virtual beam focus at the output. For low expansion ratios(1.3-20 $\times$ ) the Galilean telescope is most often employed due to its simplicity, small package size and low cost.

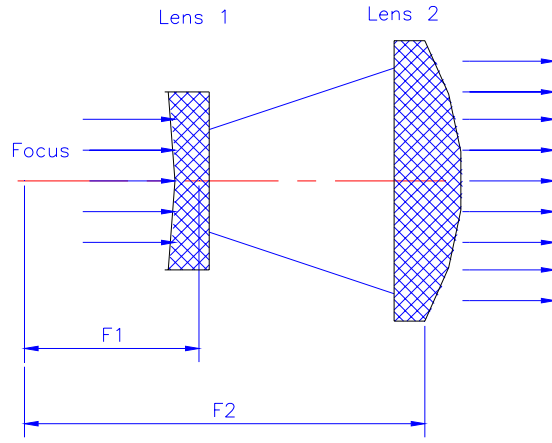


Fig. 2 Beam expander

The lens 1 focuses the laser beam from the laser generator on the front focus plane and the new beam waist  $\omega'_0$  and divergence angle  $\theta'$  can be represented as

$$\omega'_0 = \frac{\lambda F1}{\pi \omega(l)} \quad (9)$$

and 
$$\theta' = \frac{2\lambda}{\pi \omega'_0} \quad (10)$$

$$\omega(l) = \omega_0 \sqrt{1 + \left( \frac{l\lambda}{\pi \omega_0^2} \right)^2} \quad (11)$$

where  $\omega(l)$  is the radius of the beam entering the Lens 1,  $l$  is the distance between the Lens 1 and the beam waist  $\omega_0$ , and  $F1$  is the focal length of the Lens 1.

Since  $\omega'_0$  just lies on the back focus plane of the Lens 2 with a longer focal length,  $F2$ , the Gaussian beam with a beam waist  $\omega'_0$  will be collimated by the beam expander. The collimation ratio of the beam expander for a Gaussian beam is as follows

$$T = \frac{\theta}{\theta''} = T_1 \sqrt{1 + \left( \frac{l\lambda}{\pi \omega_0^2} \right)^2} \quad (12)$$

where  $T_1 = F2/F1$ . The beam waist  $\omega''_0$  and divergence angle  $\theta''$  after the beam expander are

$$\omega''_0 = \frac{\lambda}{\pi \omega_0} F2 \quad (13)$$

$$\theta'' = \frac{\theta}{T} \quad (14)$$

Substituting Equation (6) into equation (10), we have

$$\omega''_0 = T_1 \omega(l) \quad (15)$$

From above equations, we can conclude that the beam expansion ratio and the collimation ratio for a Gaussian beam depend not only on the specifications of the beam expander, but also on the laser beam parameters as well as the positions of the optical lenses.

### 3. Focusing by a Focal Lens

One of the most important specifications of focusing lens is the achievable focal spot size. If an effective diameter  $d_0$  is defined as the achievable focal spot size, which contains 86% of the focused energy, and at the edges of which the focused intensity is down to  $1/e^2=14\%$  of its peak intensity, then

$$d_0 = \frac{f\lambda}{\omega_0} \quad (16)$$

Note that this equation gives an approximation of the focused spot size. It has not taken into account the beam propagation effects, lens aberrations, and ablation thresholds of the coating materials. Substituting Equations (11) and (15) into Equation (16), the following expression is obtained

$$d_0 = \frac{f\lambda}{T_1\omega_0\sqrt{1 + \left[\frac{l\lambda}{\pi\omega_0^2}\right]^2}} \quad (17)$$

Therefore, the focus spot size can be obtained by knowing the optical system parameters.

As the Gaussian beam focuses from a lens down to a waist and then expands, there is a need to define a depth of focus. Normally, it is defined as the distance between the  $\sqrt{2}d_0$  spot size points or 2 times Rayleigh range. It can be written as

$$\Delta f = 2Z_R \approx 2\pi\lambda F^2$$

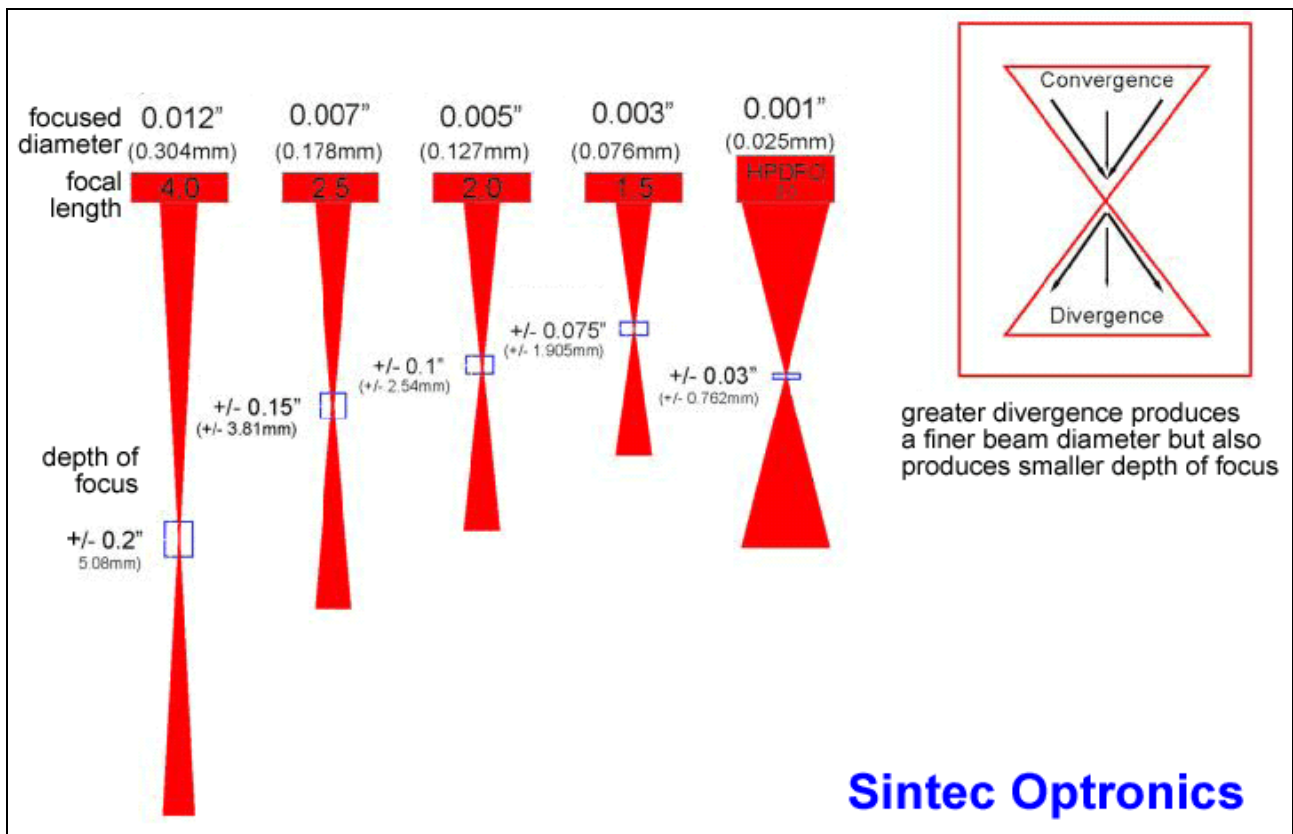
or

$$\Delta f = \frac{2f^2\theta}{D}$$

where F is the f-number of a focusing lens, which is defined as

$$F = \frac{f}{D}$$

It is concluded from above Eqs. that a lens with a longer focal length gives a greater depth of focus and a larger focus spot size than a lens with a shorter focal length. Thus the focal length of the focus lens should be selected properly according to the application requirements.



The above picture shows the focal beam diameter and depth of focus of a focusing lens in a typical CO2 laser and they are summarised in the following table:

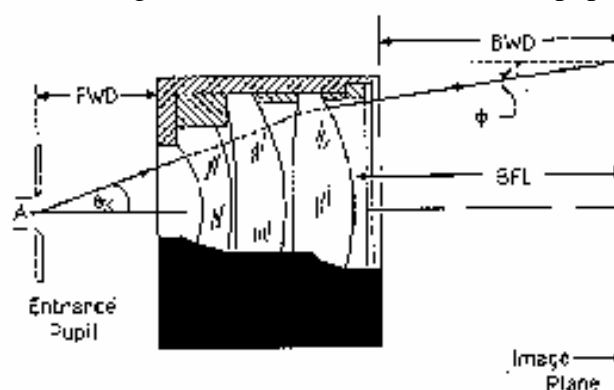
Focal length	4" (101.6mm)	2.5" (63.5mm)	2" (50.8mm)	1.5" (38.1mm)
Focal diameter	0.012" (0.304mm)	0.07" (0.178mm)	0.005" (0.127mm)	0.003" (0.076mm)
Depth of focus (+/-)	0.2" (5.06mm)	0.15" (3.81mm)	0.1" (2.54mm)	0.075" (1.905mm)

#### 4. Focusing by a f-theta Lens

The spot size will be most affected by the input laser beam diameter, divergence of the laser source, and the effective focal length of the lens system. For a diffraction limited lens coupled with a Gaussian source, the  $1/e^2$  spot size can be expressed as

$$S = 1.27\lambda \times EFL/A \quad (18)$$

where EFL is the effective focal length of the lens, A is the entrance pupil diameter.



*Fig. 3 Diagram specifying scan lens*  
*FWD: front working distance, BWD: back working distance, A: entrance pupil diameter,  $\theta$ : deflection angle*

If a single mirror system is used, the mirror is placed at the entrance pupil position and the maximum usable beam diameter is equal to the entrance pupil diameter (A). If a two mirror system is used for deflection in both the x and y directions, then the mirrors are placed on either side of the entrance pupil position and as close to each other as possible. The maximum laser beam diameter for a two axis deflection system which has been displaced a distance L from the entrance pupil is given by

$$A' = A [1 - (2L/A)\tan\theta] \quad (19)$$

where  $\theta$  is half the maximum deflection, and L is the offset distance of the mirror.

For a typical system,  $L = 17.5/2 = 8.75$  mm,  $A = 12$  mm,  $\theta = 18.40$ , and  $EFL = 236.8$  mm, therefore the theoretical spot size is  $51.6 \mu\text{m}$ .